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RELATIVISTIC PHYSICS AND GEOMETRY

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Релятивистская физика и геометрия

Проблема геометризации физики рассматривается как часть тех проблем, о которых говорится в шестой проблеме Гильберта. Эта проблема Гильберта касается математической формулировки аксиом физики. Показано, что в течение всего XX века данная проблема формировала стратегии научных исследований в теоретической физике и некоторых разделах математики, особенно в геометрии. Появление специальной и общей теорий относительности, как и геометрической теории калибровочных полей, можно рассматривать как последовательные стадии решения шестой проблемы Гильберта. Проблемой сегодняшнего дня является применение геометрической теории калибровочных полей к релятивистской ядерной физике.

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Relativistic Physics and Geometry

The problem of physics geometrization is considered as a part of the problems which is the sixth Hilbert's problem talks about. This Hilbert's problem concerns mathematical formulation of physics axioms. It is shown that for the whole XX century this problem formed scientific research strategies in theoretical physics and some mathematical topics, especially in geometry. Appearance of special and general relativities as well as the geometrical gauge field theory can be regarded as consequent stages in the sixth Hilbert's problem solution. The present-day problem consists in application of the geometrical gauge field theory for relativistic nuclear physics.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. VI HILBERT'S PROBLEM

As is known, in 1900 D. Hilbert formulated 23 problems which in his opinion the mathematicians of XX century would have to solve [1]. Among them the sixth problem pointed to necessity to state mathematical formulation of physics axioms. In this connection Hilbert proposed to construct the physics axioms on a model of the axioms of geometry. So, VI Hilbert's problem contains the problem of physics geometrization as its part.

For all XX century long this problem formed strategies of scientific researches in theoretical physics and in different mathematical topics, especially in geometry. Appearance of special and general relativities as well as the geometrical gauge field theory can be regarded as consequent stages in VI Hilbert's problem solution [2, 3].

According to the new physical theories the corresponding new geometries were appeared. New mathematics stimulated physics development and on the contrary. Minkowski 4D geometry was created for SR, Cartan's formulation of Riemannian 4D geometry arose in GR. At last the fibre bundle space geometry was formulated as extension of Cartan's geometry. It was used by me for geometrical formulation of the gauge field theory [4]. At present, this theory is the greatest extension of GR. The gauge field theory proved to be very successful in explanation of phenomena of particle physics and gravity. It permits to construct the unified theory of all fundamental interactions. Moreover, such a theory can be formulated in both usual and geometrical forms.

The problem which in the course of many years Einstein was working on now is solved in terms of the geometrical gauge field theory.

2. THE WAY TO GEOMETRICAL RELATIVITY

In one of his paper Einstein explained why he decided to look for a way to a geometrical form of gravitation theory. He called his predessors on this way German philosopher of XVIII centure I. Kant and French mathematician of XIX centure A. Poincaré.

Kant [5] established that any experiment description consists of two parts: geometry (or coordinates) and forces. At the same time, it is known that force-free or inertial motions exist, but geometry-free motions cannot exist. In the experiment description any point, particle or event are supplied with coordinates. But forces can be absent.

Poincaré [2] proposed that always it can be found such a geometry, in which any motion looks like force-free one. This idea stimulated Einstein to look for geometrical description of particle motions in a gravity field.

This situation can be expressed by the symbolic formula:

$$G = G_0 + F, \tag{1}$$

where G_0 is background geometry, i.e., rigidly given one; F is an image of acting forces; G is dynamical geometry in Einsteinian meaning, i. e., it changes in according to particle motions.

It is necessary to note that here the corresponding equations are the same in the left and right sides of formula (1).

Because the equality sign has only a symbolic sense the variable sets in the left and right sides can differ from each other. Poincaré assumed that the choice of variables depends on a convention among scientists. Consequently one can find such a geometry which is more suitable for calculations, and so any geometry can arise and be applied for solving equations.

Einstein decided to write the equations which describe a particle motion in any gravitational field as a free motion. As is known, he chose Riemannian geometry for solving of this task and obtained the equation of a geodesic line [6].

But in addition Einstein decided to clarify a physical sense of the variables in his equations. He ascertained that his motion equations did not describe the motion of any particles but only the particles having some special properties. He named such particles test bodies. They had to be a subject to gravitational influence of external field, but had not to change this field, i. e., not influence it backwards. In this way Einstein demonstrated that geometry modification in equations implied the change of physical properties of the described objects or experimental conditions which these objects are being under.

3. INNATENESS OF GEOMETRY, CONVENTIONALISM AND OVERCOMING THEM

The key problem in Poincaré–Einstein symbolic formula (1) interpretation is that: where does geometry come from?

Kant assumed that geometry was innate and arose at the same time when a child had been born. Such an answer was unacceptable for many scientists and philosophers. In spite of the fact that Kant's point of view seems strange, he proved partially right. Scientific investigations of French physiologists of XIX century showed that human equilibrium organ consists of three almost mutually perpendicular planes. As a result a man can distinguish three space dimensions since his birth.

In contrast to Kant, we have to note that really a man has only innate organs for getting of geometrical knowledge, but not this knowledge by itself. His body is a natural coordinate system and instrument for geometric constructions. But it does not contain Euclidean theorems in itself. Means of getting knowledge and knowledge itself are not one and the same.

Poincaré's point of view consisted in that: separation of right side into two parts $(G_0 + F)$ depends on us, and it is a subject to convention.

This statement is known as conventionalism and was often criticized severely by philosophers-materialists. But here is real way out of a situation. When we shall try to apply the equations to real objects behaviour in experiments, it will be clear that G_0 is a mathematical image of the device realizing the coordinate system. Thus in practice freedom of G_0 choice becomes freedom of the choice of instruments for coordinate system construction. This choice really depends on us, but it is formed by the experimental conditions, and not a convention among scientists. So, just like Kant, Poincaré was right only partially.

All the questions connected with Poincaré–Einstein formula received the most serious study in the papers of N. P. Konopleva and H. A. Sokolik in 60th of XX century [7], etc. These papers cover the problems of sovereign physical theory structure.

4. GEOMETRY IN PHYSICAL THEORY STRUCTURE

Physical theory is named sovereign if it has own means of distinguishing between true and false conclusions and, consequently, is not in need of experiments for solution of such problems. Only sovereign theory conclusions can be regarded as truth. The conditions which sovereign physical theory has to satisfy was investigated in paper [8].

Structure of axioms of such a physical theory must reflect the specific way by means of which information about external world comes into the theory. As is known, data for a physical theory usually come out of experiments. Therefore the structure of axioms of the sovereign physical theory must be closely connected with principles of experimental investigations.

For instance, the demand of result reproducibility leads to the fact that the language of theoretical physics must be the Lie group theory. Symmetry properties of the theory become defining one. If the symmetry is a global one, we have to make use of finite Lie groups. If the symmetry is a local one, we have to make use of infinite Lie groups. Representations of finite Lie groups permit us to construct the elementary particle classification. Because of representations of infinite Lie groups classification of elementary particle interactions appeared. Lagrangian theory of gauge fields on the base of infinite Lie groups representations was constructed by N. P. Konopleva in 1967 [9].

Among gauge fields gravity is associated with local translation group, which are usually named general covariant coordinate translations in 4D Riemannian space–time. Above the Lagrangian formalism and consideration of the coordinate translation group as a local gauge group permits to obtain Einsteinian theory of gravitation as the theory of one of the gauge fields. This is the only way to get usual GR as a gauge field theory. Torsions are absent in this approach.

In the gauge field theory all nongravitational fields are described by nonlinear extensions of Maxwell's equations. Electrodynamic equations coincide with the Maxwell's one.

When fundamental interactions are considered in Riemannian space–time, Einsteinian equations must be added to the other equations of the theory as local relativistic vacuum equations [10]. Thus Einsteinian equations physical sense becomes more wide.

It is remarkably that in this scheme particle motion equations can be obtained by differentiation of the field equations. It is the same situation that we have in GR. In principle, geodesic line equations could be eliminated from GR axioms because of this fact. On the other hand, trajectories of the motion of all particles carrying corresponding gauge charges in the external gauge field look like test body pathes. Therefore the Lorentz equations describing the motion of electrons in the external electromagnetic field turn into electromagnetic test body motion equations.

When free electromagnetic and gravitational fields are only present the equation system of the gauge field theory coincides with the Wheeler–Misner equations of geometrodynamics [11].

So, let us return to geometric axioms in relativistic physics.

Einstein explained essence of his geometrical approach by an imaginary experimenter being in falling lift. This experimenter has got rulers and watch, which permit him to measure segments of space and time lines. Therefore he can construct a local coordinate system in his neighbourhood. This coordinate system will be a basis of a local Euclidean space in falling lift. Origin of coordinates will coincide with a test body falling free in the external gravitational field. The equivalence principle, which is one of GR axioms, states that in given situation the experimenter does not feel gravitational field influence. Near him all events happen in just the same way as in the absence of gravity.

At the same time, other experimenter being on Earth surface outside the lift will interpret the first experimenter motion as a noninertial motion in the gravity field of Earth. Both these descriptions are right, but the first of them corresponds to local description of a motion in the accompanying coordinate system, and the second of them makes use of a global Cartesian coordinate system associated with Earth. In the second case gravity describes forces acting in Euclidean global space. This is one of realizations of Poincaré–Einstein formula. The equality sign corresponds to the equivalence principle. Two descriptions can be brought into accord with each other by identification of gravity forces with connection coefficients of 4D Riemannian space–time.

Can this method be carried to other interactions?

Throughout 30 years after creation of GR Einstein tried to unite geometrically gravity and electromagnetism. Many other authors made the same as Einstein. But at that time geometry had not yet any means for this problem solution. Cartan's formulation of Riemannian geometry of 1925 [12] described adequately the falling lift situation, but was found insufficient for new tasks.

Only in 60th of XX century fibre bundle space geometry became enough developed for its application to physics.

In 1964 it occurred to me how Einsteinian problem could be solved [13]. To this end it should be answer the question: what is a mathematical image of other measuring devices besides rulers and watch being used by the experimenter in the falling lift? My answer was following. A mathematical image of any device in any physical theory is the space of parameters measured by this device. In this space some coordinate system can be chosen like the usual space. Its origin of coordinates must coincide with the origin of usual space coordinates in which the experimenter works in the falling lift. It means that this experimenter has not only rulers and watch, but voltmeters, etc. Mathematically it leads to increase in dimensions of the space in the point where the experimenter is. At the same time, in the opinion of the external observer the experimenter moves as before in usual 4D space–time. Thus our problem reduces to carry of some many-dimensional space along lines in 4D space–time.

Such a procedure was unknown in theoretical physics. But what mathematicians could say about it? And I went to Faculty of Mechanics and Mathematics of Moscow State University.

In its library I found G. F. Laptev thesis of 1952 on imbedded manyfolds [14]. Then I learned that he leads the seminar on this problem at the Prof. P. K. Rashewski High Geometry Chair of that faculty. As it has turned out geometry which I was looking for was not exist yet but it was arising before my very eyes. Now it is named fibre bundle space geometry.

I began to attend Laptev's seminar and take part in all science conferences on differential geometry being hold in the USSR at that time. My talks were in sections of geometry applications. Unfortunately, these conferences had almost no proceedings. But I also told about applications of fibre bundle geometry to physics at conferences on theoretical physics, elementary particles and gravity as well as on philosophy and science methodology. My philosophical and methodological papers were published together with H. A. Sokolik. Geometrical formulation of the gauge field theory was given by me in 1967 and was reported on corresponding conferences in Kazan [15] and Tbilisi. Then in 1969 I had defence of thesis «Geometric Description of Interactions» at the Lebedev Institute of Academy of Sciences and, by invitation of Prof. A. M. Baldin, reported its results at the International Seminar on Vector Mesons and Electromagnetic Interactions at JINR in Dubna [16]. My thesis was written without use of a post-graduate course. It was recommended to be published by Academic counsil of LPI. In 1972 my and V. N. Popov book «Gauge Fields» (in Russian) was published by Atomizdat. For this book V. N. Popov wrote chapter IV on gauge field quantization by path integrals [17].

After that geometrical treatment of gauge fields in terms of fibre bundle space geometry [18] became generally recognized and induced development of superspace geometry in mathematics and supersymmetry gauge theories in physics. Kaluza–Klein [19] and Weyl [20] theories attracted attention of physicists again.

CONCLUSIONS

So, where did we come by axiomatization and geometrization of physics according to VI Hilbert's problem?

Unification of electrodynamics and mechanics led to creation of SR in physics and 4D Minkowski geometry in mathematics. Finite Lie groups found their wide application in physics, especially in quantum mechanics and elementary particle physics. They became the basis of classifications of elementary particles, atom and nuclear states.

But for a long time infinite Lie groups could not find their place in physics. Appearance of local coordinate translations in GR induced doubt about physical sense of this theory. Later the same doubt appeared about the gauge field theory based on local gauge symmetry groups. These groups belong to infinite Lie groups similarly to local coordinate translations in GR.

The point is that finite Lie groups have the invariants, whereas infinite Lie groups have not them. Therefore usual conservation laws vanish when symmetry of theory becomes a local one. Dynamical constants are just these numbers which physical theory produces in order to compare them with experimental data. Without conservation laws we cannot construct the dynamical constants for experiment description.

But really local symmetries should not be used for dynamical constants obtaining. In Utiyama opinion [21] they must to classify interactions between particles, but not these particles by themselves. My and Sokolik point of view consisted in that local symmetries ensure existence of gauge fields. They generate appearance of connection coefficients in a space which is only locally homogeneous. Connection coefficients are geometrical objects. What kind of physical objects is corresponding with them?

Einstein tried to geometrize electrodynamics having used additional metrics coefficients. Such coefficients arose when a space–time dimension increased (Kaluza–Klein approach) or 4D space–time metrics became asymmetric. In these cases vector-potentials of electromagnetic field became components of a metric tensor.

Weyl was first who identified electromagnetic vector-potentials with connection coefficients, but it was the only possible in terms of new geometry which Weyl constructed for this task (known as Weyl geometry [20]). Unfortunately, in this geometry correct description of Einsteinian gravity became impossible.

The problem was solved when fibre bundle space geometry arose. I identified gauge field vector-potentials with connection coefficients of fibre bundle space. 4D Riemannian space–time I turned into the base of fibre bundle space, and the space where local gauge groups were acting I identified with fibre of fibre bundle space. Gravity and nongravitational interactions became untied. Now they became acting in different spaces: gravity existed in base, and nongravitational fields were acting between fibres of fibre bundle space. GR, SR, Maxwell's electrodynamics, Wheeler–Misner geometrodynamics, Yang–Mills equations [22] were exactly reproduced in this geometry terms. Moreover, the way to unite all interactions both in usual and geometrical forms was opened.

Therefore I continued with my work in spite of sharp criticism with respect to local gauge theories from some known scientists (V. A. Fock [23], V. I. Ogiyevetsky [24], E. S. Fradkin [25], B. L. Ioffe, etc.). This skepticism was overcame by creation and use of new mathematical methods both in mathematics and physics. It was above-mentioned Lagrangian formalism for infinite Lie groups (1967, N. P. Konopleva), fibre bundle space geometry (soviet and foreign mathematicians), geometrical interpretation of gauge fields in terms of new geometry (1967, N. P. Konopleva), and quantization of gauge fields by path integrals fulfiled in 1967 by B. De Witt [26], L. D. Faddeev and V. N. Popov [27]. Renormalization of Yang–Mills fields was made in 1971 by J. C. Taylor [28] and in 1972 by A. A. Slavnov [29] (massless case) and G.' t Hooft [30] (massive case) in 1971. Discussions on fundamental questions of the quantum gauge field theory one can find in [31].

Quark models of elementaty particles [32, 33] appeared in 1964 [34]. They had played very important role in the process of gauge field theory application for elementary particle physics. Just they proved rightness of the gauge field theory in its usual form in the Minkowski space–time. Today the corresponding unified model of fundamental interactions is known as the Standard Model.

Next in turn it should be verify the gauge field theory in its geometrical form. Such experiments will be analogous to GR experiments and now they seem very complex.

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