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LEVEL DENSITY, RADIATIVE STRENGTH FUNCTIONS FROM THE $(n_{\mathrm{th}}, 2\gamma)$ REACTION AND MAIN PROPERTIES OF THE ^{96}Mo NUCLEUS

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Плотность уровней, радиационные силовые функции
из реакции $(n_{\rm th}, 2\gamma)$ и основные свойства ядра ^{96}Mo

Опубликованные данные для интенсивностей двухквантовых каскадов на 12 конечных уровней ^{96}Mo аппроксимированы для различных энергий возбуждения и дипольных первичных $E1$- и $M1$-переходов набором возможных случайных зависимостей плотности уровней и силовых функций. Средние значения этих параметров гамма-распада хорошо соответствуют основным зависимостям, выявленным из аналогичных экспериментов к настоящему времени для 42 ядер из области масс $40 \leqslant A \leqslant 200$, и не соответствуют существующим представлениям о параметрах каскадного гамма-распада компаунд-состояний ядер с высокой плотностью уровней.

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Level Density, Radiative Strength Functions
from the $(n_{\rm th}, 2\gamma)$ Reaction and Main Properties of the ^{96}Mo Nucleus

The data published on two-step cascade intensities to 12 final levels of ^{96}Mo were approximated for different energies of excitations and dipole primary $E1$- and $M1$-transitions by a set of different random dependencies of the level density and strength functions. The averaged values of these parameters of gamma-decay well correspond to main dependencies revealed by now from analogous experiments for 42 nuclei from the mass region $40 \leqslant A \leqslant 200$. And they do not correspond to the existing ideas of the cascade gamma-decay parameters of compound-nuclei with high level density.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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INTRODUCTION

The main task of experiment is determination of unknown parameters of any process (phenomenon) with the lowest deviation of obtained result from a desired and unknown value. A solution of this task can be achieved only under the following conditions:

1) obligatory experimental test of the existing and used notions of phenomenon under study, and

2) complete and careful analysis of all the sources of systematical errors of an experiment.

1. In practice, it is impossible to perform modern experiment in low energy nuclear physics without the use of some set of model notions of phenomenon under study. In this case inevitable systematical error can be minimized only by their obligatory and various test.

2. The second problem becomes the main one for experiment when direct determination of the values under study is impossible. The only possibility is solution of reverse task of mathematical analysis: determination of parameters of a function measured in experiment from its values obtained at some conditions for known (or supposed) relation between them.

Just owing to these reasons and circumstances, the obtained by now data of different experiments on level density ρ below neutron binding energy, for example, give rather ambiguous and, consequently, in some cases distorted picture of inner-nucleus processes and interactions. The best illustration of this statement is the data on ρ and radiative strength functions Γ, obtained from analysis of different gamma-ray spectra following decay of compound states (neutron resonances, in particular).

1. DIFFERENT TYPES OF INDIRECT EXPERIMENTS AND PROBLEMS OF SOLUTION OF REVERSE TASK

Nuclear excitation energy is emitted in studied nuclear reaction, as a rule, in several stages with emission of different number of nucleons (light nuclei) and/or gamma-quanta. It is possible to register in an experiment partial cross sections of emission of some single product (one-step process) or some cascade (multi-step process). Just this stage — selection of one- or two-step (multi-step) reactions — determines to high extent the reliability level of experimental data.

The difference is caused by type of dependence of the measured partial cross section on determined parameters. The measured spectrum I_1 in the case of one-step reaction is

$$I_1 \propto \rho\Gamma / \sum(\rho\Gamma). \quad (1)$$

For two-step reaction

$$I_1 I_2 \propto I_1(\Gamma / \sum(\rho\Gamma)). \quad (2)$$

Expression (2) in case of registration at the second step of reaction of gamma-quantum to only one or several nuclear levels has another shape of energy dependence of the determined ρ and Γ values than for the one-step case. And, as a consequence, considerably less coefficients of error transport $\delta(I_1 I_2)$ to errors $\delta\rho$ and $\delta\Gamma$. Just this circumstance determines potentially higher precision of two-step experiments on determination ρ and Γ.

2. PARAMETERS OF CASCADE GAMMA-DECAY OF ^{96}Mo

The first data on level density and radiative strength functions of this nucleus were obtained by Oslo group from reaction (^{3}He, α) and inelastic scattering ^{3}He [1]. Unfortunately, reliability of the obtained by them unknown up to now strongest enhancement of strength functions (at extrapolation to zero value of primary gamma-transition energy) was not grounded by analysis of experimental systematical errors. Although the necessary for such an analysis method for obtaining the calculated total gamma-spectra without random fluctuations was suggested in [2]. Its use showed that the mentioned effect can be explained even by rather insignificant systematical underestimation of the total gamma-spectra intensities which increases as decreases the energy of decaying levels of ^{96}Mo [3].

Alternative test of results [1] was performed [4] with the use of experimental data on intensities of two-step cascades to 12 final levels of this nucleus. Corresponding experiment has been performed on the thermal neutron beam in Řež. Unfortunately, the authors of the analysis did not take into account three circumstances which are very important in order to obtain reliable data on level density and radiative strength functions of the primary transitions from gamma-decay of compound-states of nuclei with high level density.

1. Each of the obtained by them experimental spectrum is superposition of two mirror symmetrical unknown distributions — intensities of cascades with primary and secondary gamma-transitions located in the same and small bins of their energies. The proof that the calculated cascade intensity for given sets of functional dependences of level density and strength functions at given energy of their primary (or secondary) transitions corresponds to analogous and unknown

experimental value can be obtained only by the use of additional experimental information. At present, the only possibility to solve this problem is known: the methods of nuclear spectroscopy for determination of the most probable dependence of cascade intensity in function of the primary transition energy [5]. Corresponding method uses only difference in shape of experimental distribution of cascade intensity at different energy of primary gamma-transitions and available spectroscopic information on decay scheme of low-lying levels of nucleus under study.

2. The summed intensity of cascades and transition to ground-state always equals 100%. Therefore, any deviation in intensity distribution of cascades terminating at arbitrary final level shifts all or unknown portion of the rest of the data. It follows from this, in particular, that the regularly observed in region $0.5B_n$ very significant enhancement of strength functions of the secondary gamma-transitions [6] distorts the value of level density obtained from cascade intensity.

3. Intensities of the two-step cascades can be reproduced with equal and minimal values of χ^2 by infinite set of different level densities and radiative strength functions. Due to nonlinearity of equations their probable values are always in limited region of magnitudes. Moreover, this region does not include, in practice for all the studied nuclei [7], the values predicted by the Fermi-gas model (for instance, in variant [8]). Therefore, any test of any sets of models (like that performed in [4]) of level density and strength functions must include the proof that the minimum of χ^2 corresponds to them. The procedure of determination of confidence level of values of desired parameters in indirect experiment is described in textbooks of mathematical statistics.

General conclusion from analysis of the two-step cascade intensities [6] and [7] is: all generally accepted ideas of level density $\rho_\lambda = D_\lambda^{-1}$ and radiative strength functions $f = \Gamma/(E_\gamma^3 D_\lambda)$ as of monotonous dependences on gamma-transition energy E_γ and excitation E_λ cannot reproduce the measured intensities of two-step cascades within precision of experiment.

Therefore, in particular, the use of model notions of level density (with their inevitable unknown systematical error) in analysis [4] for determination of strength functions from gamma-spectra guarantees appearance of equivalent systematical error in the found value. And on the contrary. Besides, coefficients of error transfer at such an analysis of experimental data have different values and in many cases can be unlimited greatly.

Practically expected systematical error at the use of methods [5] and [6] in accordance with results of analysis [9] can change values of found level density and radiative strength functions with respect to desired values as a maximum by 2–3 times if error in normalization of intensity varies in limits from –25 to $+25\%$. Moreover, corresponding errors change magnitude and sign as changing energy of gamma-transition and excitation of nucleus.

By now information on level density and radiative strength functions for even–even spherical nuclei was obtained only for ^{74}Ge, ^{114}Cd, ^{118}Sn and ^{124}Te [10]. All the above-mentioned stipulated to perform independent analysis of the data published in [4]. Unfortunately, the method of decomposition [5] of experimental intensity into components for which cascade transition with energy E_γ is the primary or secondary one and for determination of cascade population $P - i_1 = (i_1 i_2)/i_{\gamma\gamma}$ up to excitation energy ~ 5 MeV [6] requires experimental spectra. Therefore, below the analysis is performed only for the data presented in [4] in Fig. 9.

For selection of sets of level density with different parity and radiative strength functions of $E1$- and $M1$-transitions which provide the best approximation there was used a variant of the Monte-Carlo method for solution of systems on nonlinear equations used earlier in [6] and [7]. It consists in the following: initial arbitrary varied values of desired parameters are distorted by small random functions with equal probability of increasing or decreasing with respect to values of previous iteration. If a set of distorting functions decreases parameter χ^2, then initial values are changed by the distorted ones. The process realized here required not less than 20000–30000 iterations for each variant of calculation in order to achieve practically the lowest possible value of χ^2. Naturally, determination of limits of intervals of desired parameters in the case under consideration cannot be precise — it is well known that the Monte-Carlo method is ineffective at determination of low-probable events.

The obtained limits of intervals for values of random functions $\rho = \psi(E_{\rm ex})$ and $f = \phi(E_1)$ reproducing all 11 experimental spectra with practically the same minimal χ^2 are considerably wider than it can be achieved using method [5]. The width of intervals for values of parameters is strongly increased by involving in approximation of cascade intensities to levels $E_f > 1.5$ MeV.

In practice, the found width of interval for possible values of level density and strength functions is overestimated by order of magnitude relatively to potential possibilities of experiment. This conclusion follows from comparison between results obtained by means of methods [6] and [7] for the same nuclei with accounting for different quality of information accumulated in experiment in Řež and earlier (with worse equipment) in Dubna and Riga. There is not observed clearly expressed very considerable [1] (or noticeably less [4]) enhancement of strength functions at decreasing of the primary gamma-transition energy in parameters obtained by us. Some their increase, observed in approximately one quarter of cases, can be connected, most probably, with overestimation of calculated cascade intensity with the primary gamma-transitions with energy not less than 2 MeV and, correspondingly, with underestimation of intensity of the secondary gamma-transitions with the same energy. A proof of presence or absence of this strengthening cannot be obtained without the use of method [5]. One can suppose that indirectly, by considerably bigger derivative from strength functions for energy of primary transitions being by 3 MeV higher than in [1], the first possibility is more probable.

At approximation level densities of ^{96}Mo with different parity in region of neutron binding energy were taken equal and fixed on the data on neutron resonances. Densities below B_n could have different value, but their sum at $E_{\mathrm{ex}} = 2.6$ MeV was compared to known density of «discrete» levels. Variation of ratio capture cross sections of thermal neutrons for two spin channels showed that the portion $\sim 33\%$ of captures in state $J = 2$ provides for maximal quick decrease in parameter χ^2 in iterative process and simultaneously — acceptable reproduction of experimental intensities of cascades to the levels $E_f \geqslant 2.4$ MeV and ground state of this nucleus.

Experimental intensities of cascades to different final levels do not allow independent determination of ratio between thermal neutron capture cross sections [11] for both spin channels. So, almost equal total intensity of cascades to levels $E_f = 2426$ and 2438 keV ($J^\pi = 2^+$ and 5^+) cannot be reproduced in calculation at any ratio of cross sections without the assumptions on strong difference of spin dependence of levels from generally adopted law. If one does not take into account possibility of mistaken determination of spins for these levels then the hypothesis of dependence of the secondary gamma-transition intensities on structure of excited level seems to be more probable. The effect is observed in calculation, for example, in the framework of quasiparticle-phonon nuclear model [14]. Comparison between forms of the best approximating calculated intensities to the final (phonon-less) state and the first excited level (quadrupole phonon) shows their principal difference. It appears itself as clearly expressed «bump» in the centre of intensity distribution of cascades to level $E_f = 778$ keV. Such a dependence on wave function structure of cascade final level is essentially averaged at determination of $\rho = \psi(E_{\mathrm{ex}})$ and $f = \phi(E_1)$ in the framework of methods [6] and [7]. The threshold for approximation of the data in our analysis was taken equal to 0.9 by analogy with [4]. The lowest level of negative parity is $E_f = 2225$ keV, therefore strength function of the $E1$-transtions can be reliably determined only for $0.9 < E_1 < 7$ MeV.

Comparison of the experimental intensities with their most probable approximation is shown in Fig. 1 for 6 most important spectra. The sets of random functions $\rho = \psi(E_{\mathrm{ex}})$ and $f = \phi(E_1)$ obtained for the different initial data are presented in Figs. 2 and 3. These functions provide the least practically achievable value of χ^2 for all 11 experimental spectra.

As the most probable values of level density and radiative strength functions there were used their mean magnitudes. The only ground for this is the fact that the sign of error in approximation of cascade intensities in any energy interval of their primary transitions is not determined by conditions of the chosen approximation process.

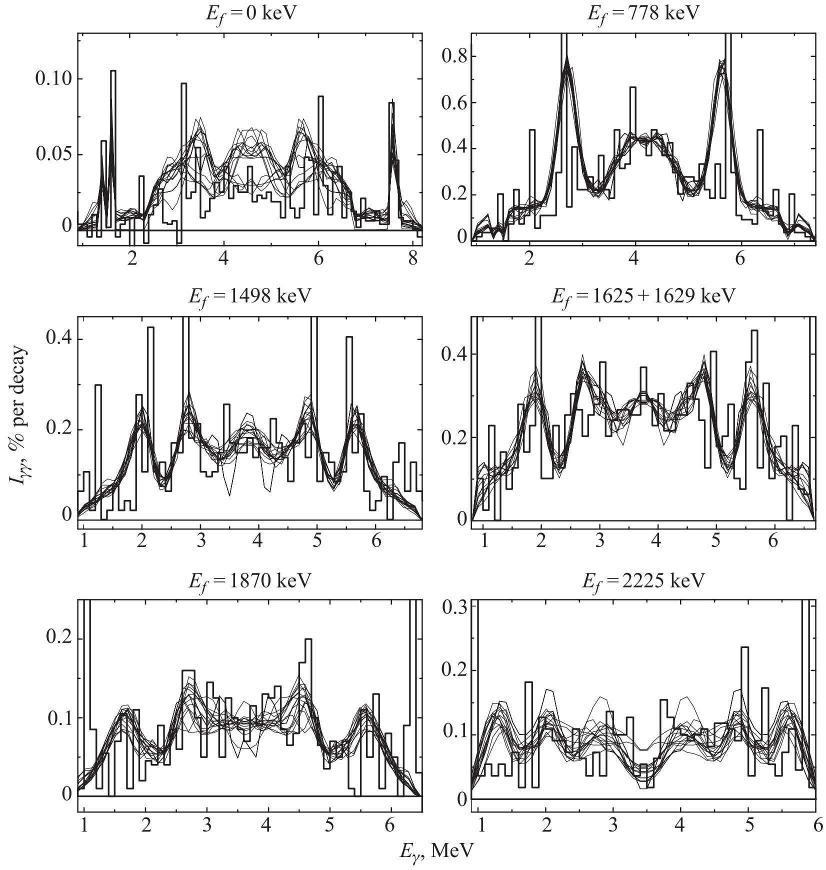

Fig. 1. Histogram — experimental intensity of two-step cascades for the levels E_f (summed over the intervals of 100 keV). Lines — variants of the calculation with random functions of level density and radiative strength functions presented in Figs. 2 and 3

The described process more effectively uses the experimental data like those presented in [4], first of all, for both throwing away any mistaken model ideas of level density and strength functions and for revealing of considerable systematical errors in corresponding data of the other experiments. The use of any assumed functions $\rho = \psi(E_{\rm ex})$ and $f = \phi(E_1)$ as the initial data for iterative process allows one to search for arbitrary random solutions and to compare the values of χ^2/f for them.

The values obtained for given nucleus in iterative process allow one undoubtedly to reject, for example, ideas of Fermi-gas model for level density as

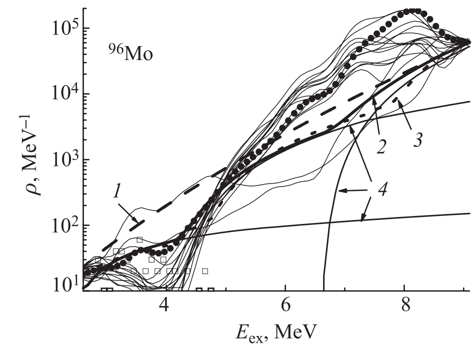

Fig. 2. Line *1* — model values [10], set of thin curves — the best random functions of the density of intermediate cascade levels, reproducing $I_{\gamma\gamma}$ with practically the same least values χ^2. Points — their mean value. Line *2* — approximation by model [18] with parameter g, depending on shell inhomogeneities of one-particle spectrum; line *3* — the same for $g = \mathrm{const}$. Lines *4* — partial densities of two-, four- and six-quasiparticle levels. Squares — density of known levels from evaluated decay scheme [19]

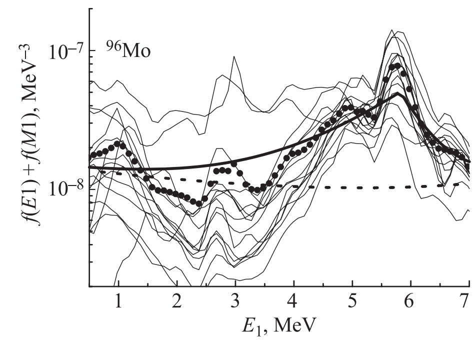

Fig. 3. Thin curves — the best random functions reproducing $I_{\gamma\gamma}$ (Fig. 1) with practically the same smallest values χ^2. Points — their mean value. Solid curve — the best approximation by model [17], dotted curve — contribution in strength function of model [20]

not corresponding with experiment. Analogous conclusion follows as well for «low-energy tail» of radiative strength functions [1]. In spite of considerable and inevitable uncertainty of their obtained values, the data presented in Figs. 2 and 3 are high-informative and reliable enough to get conclusions on properties of nucleus ^{96}Mo which appear themselves in process of cascade gamma-decay of its compound state.

3. PICTURE OF CASCADE GAMMA-DECAY OF COMPOUND STATE OF ^{96}Mo

Practical absence of somewhat intense cascades with $E2$-transitions in studied by now ≈ 50 nuclei from the mass region $40 \leqslant A \leqslant 200$ permits us not to consider any multipolarities higher than dipole in analysis like [6] and [7]. The presence of mixtures of multipolarites of $M1$- and $E2$-transitions found, for example, at depopulation of low-lying levels undoubtedly changes (in limits of selection rule on multipolarity) the values of obtained experimentally strength functions. But this phenomenon requires to be taken into consideration only at possible development of modern model notions on this account on the basis of the data [6] and [7].

If one does not take into account potential possibility of considerable uncertainties at normalization of two-step cascade intensities by authors [4] then the most probable error in determination of both level densities and strength functions is caused in our analysis only by the use of mistaken hypothesis by Axel–Brink [12, 13]. This was pointed out in [6] by enhancement of cascade population of levels of any nuclei in the region below breaking threshold of the second Cooper pair of nucleons. This error can be completely rejected only at accurate accounting for functional dependence of strength functions not only on energy of dipole gamma-transition but also on energy of excited level practically in all diapason of neutron binding energy.

In practice, real extent of decrease of error is determined only by volume of the experimental data on cascade population of levels lying above $\approx 0.5B_n$. Due to this reason one can expect from comparison of the data for the same nuclei from [6] and [7] that the level density of ^{96}Mo in Fig. 2 is overestimated in the region $\sim 3-5$ MeV or more wide by 2–3 times. Strength functions of the primary gamma-transitions with energy $\sim 4-6$ MeV in Fig. 3 are correspondingly underestimated. In spite of this error, approximation of level density by the model of partial density of n-quasiparticle excitations in variants [15] and [16] with constant coefficient of collective enhancement ($K_{\rm coll} = 6.7$ and 3.5, respectively) and sum of strength functions by semi-phenomenological model [17] shows that the process of cascade gamma-decay of compound-state of this nucleus obeys regularities, earlier obtained by us: both level density and strength functions are determined by coexistence and interaction of usual and superfluid phases of nuclear matter. Comparison of parameters of approximating function for 5 even–even compound spherical nuclei of middle mass is performed in table.

Overestimation of coefficient κ and underestimation of P_1 are to high extent stipulated only by the use at approximation of the experimental two-step cascade spectra instead of the potentially possible use of intensity of cascades in function on their primary transition energies. In this case, the error significantly

Parameters of approximation of level density and strength functions for different nuclei: coefficient of change in square of nuclear temperature κ and contribution w of model [20] in summed strength function. E_1 — position of local peak and its amplitude P_1 (multiplied by 10^{-7}), α — rate of decrease of local peak amplitude when primary transition energy decreases. Parameter of level density a, coefficient of collective enhancement of level density $K_{\rm coll}$ and thresholds U of breaking up of the second and third Cooper pairs in variant of approximation [16], which takes into account shell inhomogeneities of one-particle spectrum

Parameter	^{74}Ge	^{96}Mo	^{114}Cd	^{118}Sn	^{124}Te
κ	0.14(6)	0.41(16)	0.18(9)	0.04(25)	0.18(3)
w	0.25(4)	0.16(3)	0.10(4)	0.01(4)	0.56(6)
E_1, MeV	5.4(1)	5.8(2)	5.7(1)	5.0(1)	7.3(1)
P_1	5.3(2)	4.0(6)	9.6(36)	8.2(6)	7.2(7)
α, MeV^{-1}	0.59(35)	0.74(21)	0.89(6)	0.90(8)	0.80(9)
a, MeV^{-1}	9.96	11	13	13.3	15.6
$K_{\rm coll}$	17	6.7	13	4.5	15
U_2, MeV	5.9	3.7	3.8	4.7	2.8
U_3, MeV	8.8	6.5	5.9	4.3	6.6

increases owing to involving in analysis of cascades to final levels with energy higher than ~ 1 MeV: relative portion of intensity of cascades with low-energy secondary gamma-transitions considerably increases due to increasing number of intermediate cascade levels with energy more than ~ 1 MeV.

It was impossible to achieve complete correspondence between experimental level density and its approximation in this nucleus as in some cases obtained earlier (Fig. 2). If one excludes from consideration ordinary experimental errors then there can be the following potential sources of systematical errors of the data presented in Figs. 2 and 3:

1. The use of the Axel–Brink hypothesis instead of unknown strength functions $f(E_\gamma, E_f)$ which depend on structure of wave function of the excited level E_f;

2. Branching coefficients at decay of any intermediate cascade levels depend on way of their excitation;

3. Excitation probability of different intermediate levels depends on structure of decaying compound state (i. e., all totality of the primary levels presents itself as two or several independent or weakly interacting systems, and they cannot be described by the only strength function);

4. Density of observed neutron resonances is considerably less than that of all possible levels with the value of J^π;

5. Coefficients of vibrational enhancement of level density rather essentially change with the change of nuclear excitation energy and so on.

CONCLUSION

One can conclude to a precision of possibilities mentioned above that the process of cascade gamma-decay allows one to study dynamics of interaction between superfluid and usual phases of nuclear matter. In particular, joint model reproduction in the framework of such an approach of both level density and probable emission of gamma-quanta opens potential possibility for determination of correlation functions of Cooper pairs of nucleons in heated nuclei.

The basis for obtaining this information is the least as compared with other methods systematical errors of the experiment and maximal reduction of their influence on mathematically correct method of data treatment. Alternative variants of analysis of two-step cascade intensities and spectra of gamma-transitions in nuclear reactions give values of level density and radiative strength functions with systematical error which is many times larger than that achieved in the framework of [6].

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