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FISSION AND NUCLEAR LIQUID-GAS  
PHASE TRANSITION

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Деление и ядерный фазовый переход жидкость–газ

Рассматривается зависимость жидко-капельного барьера деления от температуры, при этом критическая температура для фазового перехода жидкость–газ является параметром. Экспериментальные данные для вероятности деления возбужденного ядра  $^{188}\text{Os}$  сравниваются с расчетом. Вычисления проведены в рамках статистической модели. Получено, что критическая температура для ядерного фазового перехода жидкость–газ больше, чем 16 МэВ.

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Fission and Nuclear Liquid–Gas Phase Transition

The temperature dependence of the liquid-drop fission barrier is considered, the critical temperature for the liquid–gas phase transition in nuclear matter being a parameter. Experimental and calculated data on the fission probability are compared for highly excited  $^{188}\text{Os}$ . The calculations have been made in the framework of the statistical model. It is concluded that the critical temperature for the nuclear liquid–gas phase transition is higher than 16 MeV.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

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## 1. INTRODUCTION

The critical temperature for the liquid–gas phase transition is a crucial characteristic related to the nuclear equation of state. There are many calculations of  $T_c$  for finite nuclei. In [1–5], it is done by using a Skyrme effective interaction and the thermal Hartree–Fock theory. The values of  $T_c$  were found to be in the range 10–20 MeV depending upon the chosen interaction parameters and the details of the model. In [6, 7] the thermodynamic properties of nuclei are considered employing the semi-classical nuclear model, based on the Seyler–Blanchard interaction. The value of critical temperature is estimated to be  $T_c = 16.66$  MeV.

As the temperature of a nucleus increases, the surface tension decreases and then *vanishes* at  $T_c$ . For temperatures below critical, two distinct nuclear phases coexist — liquid and gas. Beyond  $T_c$  there is not two phase equilibrium, only nuclear vapor exists.

The main source of the experimental information for  $T_c$  is the yield of intermediate mass fragments. In some statistical models of nuclear multi-fragmentation the shape of the IMF charge distribution,  $Y(Z)$ , is sensitive to the ratio  $T/T_c$ . It was noted in the earlier papers that the fragment charge distribution is well described by the power law,  $Y(Z) \sim Z^{-\tau}$  [8], as predicted by the classical Fisher droplet model for the vicinity of the critical point [9]. In [8] the critical temperature was estimated to be  $\sim 5$  MeV simply from the fact that the IMF mass distribution is well described by a power law for the collision of  $p$  (80–350 GeV) with Kr and Xe. In the paper [10] the experimental data were gathered for different colliding systems to get the temperature dependence of the power law exponent. The temperature was derived from the inverse slope of the fragment energy spectra in the range of the high-energy tail. The minimal value of  $\tau$  was obtained at  $T = 11$ – $12$  MeV, which was claimed as  $T_c$ . The later data smeared out this minimum. Moreover, it became clear that the «slope» temperature for fragments does not coincide with the thermodynamic one, which is significantly smaller. A more sophisticated use of Fisher’s model has been made in [11]. The model is modified by including the Coulomb energy release, when a particle moves from the liquid to the vapor. The data for multi-fragmentation in  $\pi$  (8 GeV/c) + Au collisions were analyzed. The extracted critical temperature

was  $(6.7 \pm 0.2)$  MeV. The same analysis technique was applied for collisions of Au, La, Kr (at 1.0 GeV per nucleon) with a carbon target [12]. The extracted values of  $T_c$  are  $(7.6 \pm 0.2)$ ,  $(7.8 \pm 0.2)$  and  $(8.1 \pm 0.2)$  MeV, respectively.

Significantly higher critical temperature,  $(16.6 \pm 0.86)$  MeV, was obtained in [13] by semi-empirical analysis of the data for the «limiting temperatures» of fragmenting systems. Authors of [13] interpreted the obtained value as  $T_c$  for the symmetric nuclear matter.

Having in mind the shortcomings of Fisher's model [14, 15] we have estimated the nuclear critical temperature in the framework of the statistical multifragmentation model, SMM [16]. This model describes well the different properties of thermal disintegration of target spectators produced in collisions of relativistic light ions. The intermediate mass fragment (IMF) yield depends on the contribution of the surface free energy to the entropy of a given final state. The surface tension coefficient of hot nuclei depends on the critical temperature. The comparison of the measured and calculated IMF charge yields provides a way to estimate  $T_c$ . It was found from the analysis of the fragment charge distributions for the  $p$  (8.1 GeV)+Au reaction that  $T_c = (20 \pm 3)$  MeV [17]. In the next paper by the FASA collaboration [18] the value  $T_c = (17 \pm 2)$  MeV was obtained from an analysis of the same data using a slightly different separation of the events.

Thus, the different experimental estimations of the critical temperature from fragmentation data are very controversial. This is a reason to look for other observables which are sensitive to the critical temperature for the liquid–gas phase transition. It was suggested in [19] to analyze the temperature dependence of the fission probability to estimate  $T_c$ . Note, that Silva et al. [20] explain why the power law used in the Fisher droplet model gives a spurious value for  $T_c$ .

## 2. TEMPERATURE DEPENDENCE OF FISSION BARRIER

The fissility of heavy nuclei is determined by the ratio of the Coulomb and surface free energies: the larger the ratio, the smaller the fission barrier. As the temperature approaches the critical one from below, the surface tension (and surface energy) gradually decreases, and the fission barrier becomes lower. Thus, the measurement of fission probabilities for different excitation energies allows an estimate of how far the system is from the critical point. Temperature effects in the fission barrier have been considered in a number of theoretical studies based on different models (see, e.g., [1, 21–26]). The effect is so large for hot nuclei that the barrier vanishes, in fact, at temperatures of 4–6 MeV for critical temperature  $T_c$  in the range 15–18 MeV.

In terms of the standard liquid–drop conventions [27], the fission barrier can be represented as a function of temperature by the following relation:

$$\begin{aligned} B_f(T, T_s) &= E_s(T_s) - E_s^0(T) + E_c(T_s) - E_c^0(T) = \\ &= E_s^0(T) [(B_s - 1) + 2x(T) \cdot (B_c - 1)]. \end{aligned} \quad (1)$$

Here  $B_s$  is the surface (free) energy at the saddle point in units of surface energy  $E_s^0(T)$  of a spherical drop;  $B_c$  is the Coulomb energy at the saddle deformation in units of Coulomb energy  $E_c^0(T)$  of the spherical nucleus;  $T_s$  and  $T$  are temperatures for the saddle and ground state configurations. For the surface energy and the fissility parameter  $x(T)$ , one can write [21]:

$$E_s^0(T) = E_s^0(0) \frac{\sigma(T)}{\sigma(0)} \left[ \frac{\rho(0)}{\rho(T)} \right]^{2/3}, \quad x(T) = \frac{E_c^0(T)}{2E_s^0(T)} = x(0) \frac{\rho(T)\sigma(0)}{\rho(0)\sigma(T)}, \quad (2)$$

where  $\sigma(T)$  and  $\rho(T)$  are the surface tension and the mean nuclear density for a given temperature. Equation (1) can be written as

$$B_f(T, T_s) = B_f(T_s) + \Delta B_f, \quad (3)$$

where  $\Delta B_f = E_s^0(T_s) - E_s^0(T) + E_c^0(T_s) - E_c^0(T)$ . Here  $B_f(T_s)$  is fission barrier calculated under assumption that  $T_s = T$ . In that case the values  $B_s$  and  $B_c$  are determined by the deformation at the saddle point, which depends on the fissility parameter  $x(T)$ . These quantities were tabulated by Nix [27] for the full range of the fissility parameter. The value of  $\Delta B_f$  is determined by the surface and Coulomb energies of a spherical drop, and can be easily calculated. For  $\sigma(T)$  we use the approximation

$$\sigma(T) = \sigma(0) \left[ \frac{T_c^2 - T^2}{T_c^2 + T^2} \right]^{5/4}. \quad (4)$$

This equation obtained in [28] was devoted to the consideration of thermodynamic properties of a plane interface between liquid and gaseous phases of nuclear matter in equilibrium. This parameterization is successfully used by the SMM for describing the multi-fragment decay of hot nuclei. Figure 1 shows the different approximations used in the literature for the surface tension coefficient as a function of  $T/T_c$ .

Curve number 2 was calculated in the framework of semi-classical model, based on the Seyler–Blanchard interaction [7]. An analytical expression for  $\sigma(T)$  obtained in this paper is the following:

$$\sigma(T) = \sigma(0) \left( 1 + 1.5 \frac{T}{T_c} \right) \left( 1 - \frac{T}{T_c} \right)^{1.5}. \quad (5)$$

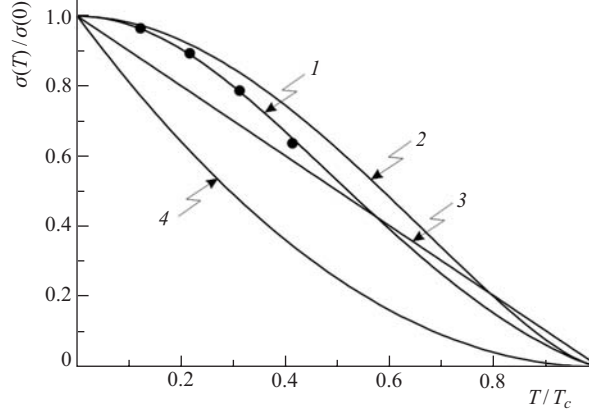


Fig. 1. The calculated coefficient of the surface tension as a function of  $T/T_c$ : curves 1 and 2 are obtained according to Eqs. (4) and 5), curves 3, 4 are for linear and quadratic parameterizations of  $\sigma(T)$ . The symbols are taken from [1]

Two other parameterization of  $\sigma(T)$  are also presented: linear  $\sim (1 - T/T_c)$ , which is used in the analysis with the Fisher droplet model [11,12], and quadratic  $\sim (1 - T/T_c)^2$  [29].

In accordance with [27], the expressions for  $E_s^0(0)$  and  $x(0)$  are taken to be

$$E_s^0(0) = 17,9439\gamma \cdot A^{2/3} \text{ MeV}, \quad x(0) = \frac{Z^2/A}{50.88\gamma}, \quad (6)$$

where  $\gamma = 1 - 1.7826 \left[ \frac{N-Z}{A} \right]^2$ . Sauer et al. [1] investigated the thermal properties of nuclei by using the Hartree–Fock approximation with the Skyrme force. The equation of state was obtained, which gives the critical temperature  $T_c \approx 18$  MeV for finite nuclei. The temperature dependence of the mean nuclear density was found to be  $\rho(T) = \rho(0)(1 - \alpha T^2)$ , where  $\alpha = 1.26 \cdot 10^{-3} \text{ MeV}^{-2}$ . In the following we shall use this finding for  $\rho(T)$ .

Figure 2 shows the relative values of the fissility parameter  $x(T)$  for  $^{188}\text{Os}$  calculated as a function of reduced temperature  $T/T_c$ . This nucleus has been chosen since the results can be compared with well-known experimental data for this nucleus [30]. The calculations are performed for the different versions of  $\sigma(T)$  mentioned above. A drastic change of nuclear fissility is expected even halfway to the critical point.

Figure 3 displays the calculated value of the liquid–drop fission barrier for  $^{188}\text{Os}$  as a function of relative temperature. Virtually, the barrier vanishes for  $T > 0.4T_c$  if the surface tension is taken according to (4) and (5). For the linear and quadratic approximations of  $\sigma(T)$  the reduction of the fission barrier with temperature is much faster.

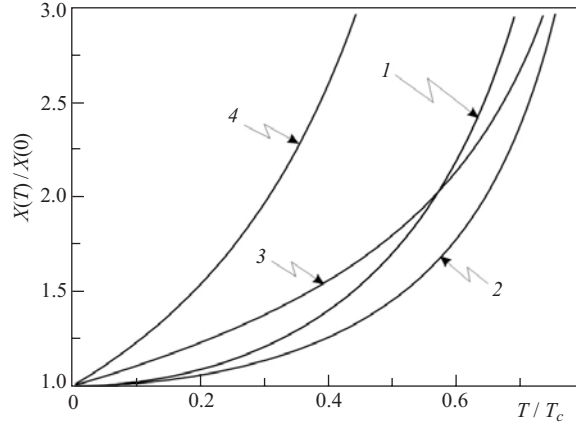


Fig. 2. Relative value of fissility parameter, calculated for  $^{188}\text{Os}$  as a function of relative temperature for different parameterization of surface tension. Meaning of the curves is explained in caption of Fig. 1

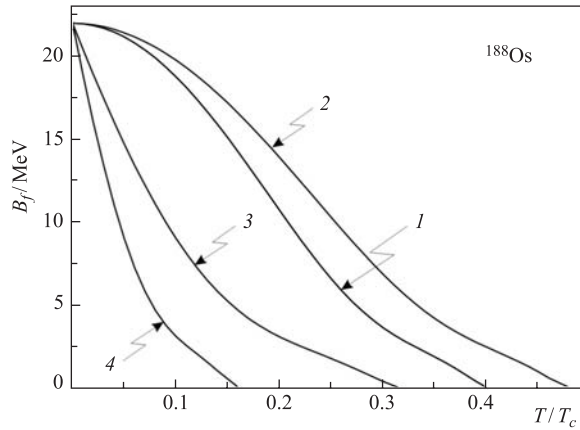


Fig. 3. Temperature dependence of liquid-drop fission barrier for  $^{188}\text{Os}$ . The meaning of the curves is explained in caption of Fig. 1

### 3. THE ESTIMATION OF FISSION PROBABILITY

In this section we analyze the experimental data on the fission probability of  $^{188}\text{Os}$ , produced in collisions  $^4\text{He}+^{184}\text{W}$  [30]. The excitation energy of the compound nucleus created at the highest beam energy is 117 MeV. The shell and

pairing effects are predicted to disappear for such a hot nucleus; therefore the fission barrier is expected to be the liquid-drop one. This barrier is temperature-dependent. Comparison of the measured and model calculated fission probabilities provides a way to estimate the critical temperature  $T_c$ .

Experimentally the fission probability  $W_f$  can be found from the measured fission cross section  $\sigma_f$ :

$$W_f = \sigma_f / \sigma_R, \quad (7)$$

where  $\sigma_R$  is total reaction cross section. The main decay mode of the compound nucleus in  ${}^4\text{He}+{}^{184}\text{W}$  collisions is the sequential emission of neutrons. For the highest excitation energy the mean number of emitted neutrons is 11–12. The mean fission probability during a neutron cascade of  $x$  steps can be calculated by the following equation:

$$W_f = 1 - \prod_{i=1}^x \left[ 1 - \frac{\Gamma_f(A_i, Z_i, E_i^*)}{\Gamma_{\text{tot}}(A_i, Z_i, E_i^*)} \right]. \quad (8)$$

The ratio  $\Gamma_f/\Gamma_{\text{tot}}$  is the relative fission width for the  $i$ -step of the cascade. According to the statistical model [31] the value of  $\Gamma_f$  is calculated as

$$\Gamma_f(E_i^*, I_i) = \frac{1}{2\pi \cdot \rho(U_i)} \int_0^{U_i - B_{fi}} \rho_S(U_i - B_{fi} - \varepsilon) d\varepsilon. \quad (9)$$

Here  $U$  is the thermal part of excitation energy  $E^*$ ,  $\rho(U)$  is the level density, the index  $s$  is used for the saddle configuration. It is natural to use in (3) the temperature-dependent fission barrier as has been done in [21–26]. The problem was considered also in [32]. The neutron width is given by the following equation [33]:

$$\Gamma_n(E_i^*, I_i) = \frac{2(2S_n + 1)m_n}{\pi^2 h^3 \rho_i(U_i)} \int_0^{U_i - B_{ni}} \sigma_n(E_n) \rho_i(U_i - B_{ni} - E_n) E_n dE_n. \quad (10)$$

Here  $B_{ni}$ ,  $E_n$ ,  $S_n$  are binding, kinetic energies and spin of the neutron,  $\sigma_n(E_n)$  is the neutron capture cross section for the inverse reaction. The contribution of charged particle evaporation is on the level of several percent of  $\Gamma_{\text{tot}}$ . Nevertheless it has been taken into account. For level density  $\rho(U)$  the Fermi-gas model is used.

Figure 4 presents the comparison of the data for fissility of  ${}^{188}\text{Os}$  as a function of excitation energy [30] with calculations under the assumption that the surface tension is described by Eq.(4). The critical temperature is a parameter that can



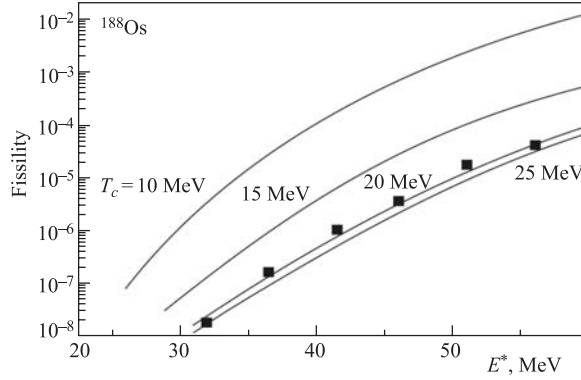


Fig. 4. Fission probability of  $^{188}\text{Os}$  as a function of the excitation energy: dots are data [30], curves are calculated assuming different values of critical temperature. Surface tension is taken according to Eq. (4)

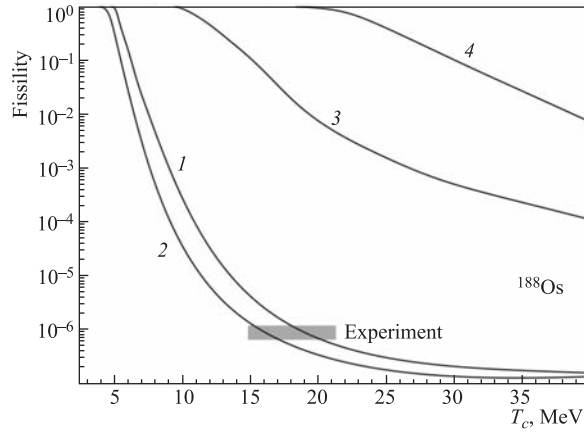


Fig. 5. Fission probabilities for  $^{188}\text{Os}$  at excitation energy 40 MeV. The calculated values (curves) are given as a function of the assumed critical temperature. Different parameterizations of surface tension are used (see Fig. 1). The experimental value is shown by the horizontal band

be found from the best fit. It is done for the highest excitation energy available, where the temperature dependence of the fission barrier is more prominent.

The result is demonstrated in Fig. 5. Different calculations are presented, which have been done using all the parameterization of the surface tension men-

tioned above. It seems clear that the linear and quadratic approximations for  $\sigma(T)$  should be excluded as unrealistic. Fission probabilities, calculated with Eqs. (4) and (5) fall rather fast with increasing the critical temperature. They are crossing the experimental band giving the following values of critical temperature:  $T_c \approx (19.5 \pm 1.2)$  MeV in the first case, and at  $T_c \approx (16.5 \pm 1.0)$  MeV for the use of Eq. (5). This is in accordance with the value of the critical temperature obtained by the FASA collaboration from multi-fragmentation data. These values are only slightly changed when the shell effect is taken into account for the last steps of the neutron cascade.

#### 4. CONCLUSION

Critical temperature for the nuclear liquid–gas phase transition has been estimated from the fission probability of the highly excited nucleus  $^{188}\text{Os}$ . Analysis is made under different assumptions about the temperature dependence of nuclear surface tension. The results presented here provide strong support for the value  $T_c \geq 16$  MeV.

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