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A. P. Severyukhin, V. V. Voronov, I. N. Borzov,
Nguyen Van Giai*

FINITE RANK SEPARABLE APPROXIMATION FOR SKYRME INTERACTIONS: SPIN–ISOSPIN EXCITATIONS

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*Institut de Physique Nucléaire, CNRS-IN2P3 and University Paris-Sud,
91405 Orsay, France

Северюхин А. П., Воронов В. В., Борзов И. Н., Нгуен Ван Джай E4-2012-136
Сепарабельное приближение для взаимодействия Скирма:
спин-изоспиновые возбуждения

Сепарабельный подход, построенный на приближении случайных фаз для взаимодействия Скирма, применен к изучению зарядово-обменных мод ядерных возбуждений. В расчетах учтена связь между одно- и двухфононными компонентами волновой функции. На примере спин-дипольного резонанса в ^{132}Sn показано, что сепарабельное приближение применимо для этих мод возбуждений. Возможности метода продемонстрированы на примере изучения связи простых и сложных конфигураций на период β-распада ^{78}Ni.

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Severyukhin A. P., Voronov V. V., Borzov I. N., Nguyen Van Giai E4-2012-136
Finite Rank Separable Approximation for Skyrme Interactions:
Spin–Isospin Excitations

A finite rank separable approximation for the quasiparticle random phase approximation with the Skyrme interactions is applied for the case of charge-exchange nuclear modes. The coupling between one- and two-phonon terms in the wave functions are taken into account. It has been shown that the approximation reproduces reasonably well the full charge-exchange RPA results for the spin-dipole resonances in ^{132}Sn. As an illustration of the method, the phonon–phonon coupling effect on the β-decay half-life of ^{78}Ni is considered.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

A study of the charge-exchange nuclear modes is an interesting problem not only from the nuclear structure point of view, but it is very important for the nuclear astrophysics applications. One of the successful tools for these studies is the quasiparticle random phase approximation (QRPA) with the self-consistent mean field derived from the Skyrme interaction, see, for example, [1–4]. These QRPA calculations allow one to describe the properties of the ground states and excited states using the same energy density functional.

A comparison of such calculations with recent experimental data [5] demonstrates that above-mentioned model cannot reproduce correctly the strength distribution of the spin–isospin resonances. It would be desirable to extend the description beyond the QRPA scheme in order to include damping effects observed experimentally [6–8]. Using the Skyrme functional within the RPA, such attempts in the past [9, 10] allow one to understand the damping of charge-exchange resonances and their particle decay. Recently, the damping of the Gamow–Teller (GT) mode was investigated using the Skyrme RPA plus particle-vibration coupling [11]. However, the size of the configuration space increases very rapidly and one has to study only a limited number of typical cases.

It is much simpler to include a coupling in QRPA calculations if one uses separable forces [7, 12]. This idea stimulated us to develop the finite rank separable approximation (FRSA) for the Skyrme interactions [13, 14] that enables one to perform calculations in the large configuration space. Applications of our method to study the low-lying states and giant resonances within the QRPA and beyond are given in [13–16]. Recently, we have proposed an extension of our approach for the charge-exchange nuclear excitations [17].

Here, we describe briefly our method for the charge-exchange excitations and present an extension by taking into account the coupling between one- and two-phonon terms in the wave functions of excited states. Before to investigate effects of the coupling, we check that the FRSA is good enough to reproduce main characteristics of such nuclear modes. Finally, this report gives an illustration of the approach for the β-decay half-life of ^{78}Ni.

1. METHOD OF CALCULATIONS

The starting point of the method is the HF–BCS calculation [18] of the parent ground state, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the Skyrme HF Hamiltonian on a harmonic oscillator basis. As effective interactions, the Skyrme interaction are used in the particle–hole (p–h) channel, and the pairing correlations are generated by a density-dependent zero-range force [14, 19].

The residual interaction $V_{\rm res}$ can be obtained as the second derivative of the energy density functional with respect to the densities. Following the paper [13], we simplify $V_{\rm res}$ by approximating it by its Landau–Migdal form. For Skyrme interactions all Landau parameters with $l > 1$ are zero. We keep only the $l = 0$ terms in $V_{\rm res}$ and the Landau parameter expressions in terms of the Skyrme force parameters can be found in [14, 20]. The Coulomb and spin-orbit residual interactions are dropped. The matrix elements of $V_{\rm res}$ can be written as the separable form in the angular coordinates. After integrating over the angular variables, we use the N-point integration Gauss formula for the radial integrals. Thus, the residual interaction can be expressed as the sum of N terms in FRSA for the Skyrme residual interaction [13, 14, 17].

We introduce the neutron–proton phonon creation operators

$$Q^{+}_{JMi} = \sum_{j_n j_p} \left(X^{Ji}_{j_n j_p} A^{+}(j_n j_p; JM) - (-1)^{J-M} Y^{Ji}_{j_n j_p} A(j_n j_p; J-M) \right), \quad (1)$$

$$A^{+}(j_n j_p; JM) = \sum_{m_n m_p} \langle j_n m_n j_p m_p \mid JM \rangle \alpha^{+}_{j_n m_n} \alpha^{+}_{j_p m_p}, \quad (2)$$

where J denotes the total angular momentum and M is its z-projection in the laboratory system. α^{+}_{jm} (α_{jm}) is the quasiparticle creation (annihilation) operator and jm denote the quantum numbers $nljm$. One assumes that the ground state is the phonon vacuum $\mid 0\rangle$ and the one-phonon excited states are $Q^{+}_{JMi} \mid 0\rangle$. Solutions of the charge-exchange QRPA equations yield the energies ω_{Ji} and the amplitudes $X^{Ji}_{j_n j_p}$, $Y^{Ji}_{j_n j_p}$ of the excited states. The excitation energies with respect to the parent ground state are given by

$$\Omega^{\mp}_{Ji} = \omega_{Ji} \mp (\lambda_n - \lambda_p), \quad (3)$$

in both the $T_{\mp}$ channels. The quantities λ_n and λ_p are the neutron and the proton chemical potentials, respectively. The FRSA enables one to obtain the energies ω_{Ji} as the roots of the secular equation (the matrix dimension never exceeded $4N \times 4N$), and the phonon amplitudes can be calculated by performing the partial summations of the secular matrix [17].

In the next stage, we construct the wave functions from a linear combination of one-phonon and two-phonon configurations

$$\Psi_{\nu}(JM) = \left(\sum_i R_i(J\nu) Q^{+}_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^{+}_{\lambda_1 \mu_1 i_1} \bar{Q}^{+}_{\lambda_2 \mu_2 i_2} \right]_{JM} \right) |0\rangle. \quad (4)$$

$\bar{Q}^{+}_{\lambda\mu i}|0\rangle$ is the QRPA state having energy $\bar{\Omega}_{\lambda i}$. Non-charge-exchange QRPA is performed in the same way as charge-exchange QRPA [14].

The normalization condition for the wave functions (4) is

$$\sum_i R_i^2(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 = 1. \tag{5}$$

Using the variational principle, we obtain a set of linear equations

$$(\Omega_{\lambda i} - E_\nu) R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0, \tag{6}$$

$$(\Omega_{\lambda_1 i_1} + \bar{\Omega}_{\lambda_2 i_2} - E_\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) + \sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) R_i(J\nu) = 0. \tag{7}$$

Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

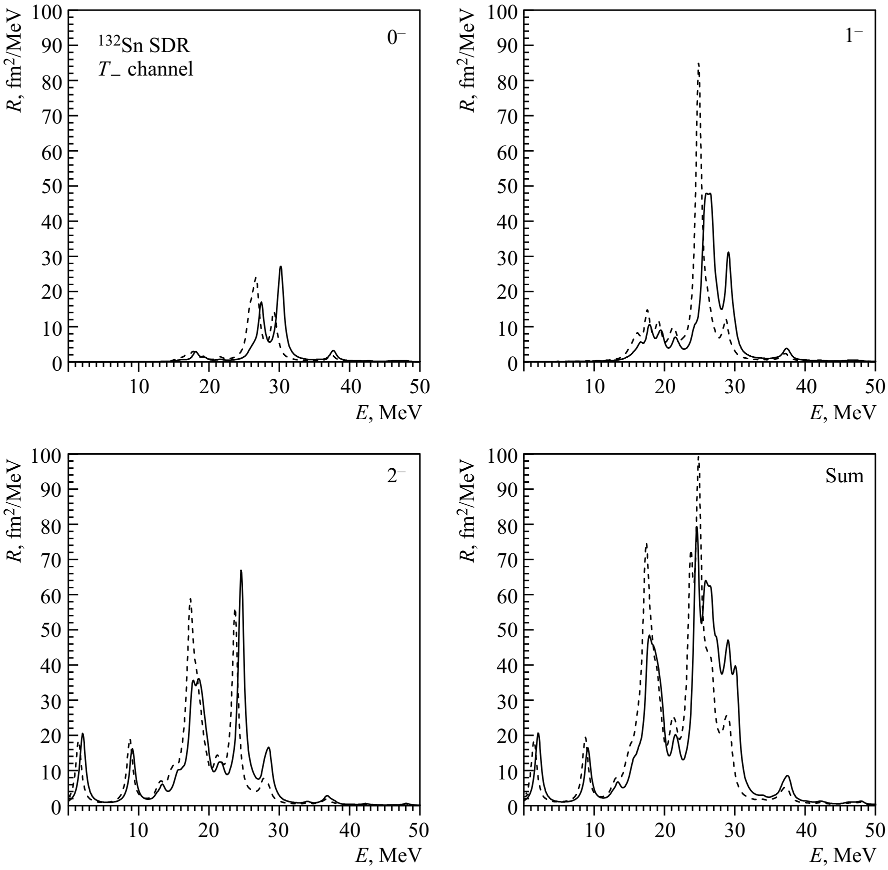
$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0| Q_{\lambda i} H \left[Q^{+}_{\lambda_1 i_1} \bar{Q}^{+}_{\lambda_2 i_2} \right]_J |0\rangle. \tag{8}$$

Equations (6), (7) have the same form as the equations of the quasiparticle–phonon model [7, 12], but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

2. RESULTS OF CALCULATIONS

First, we evaluate the accuracy of the FRSA by comparing results obtained using this separable approximation with those from a full treatment of the Skyrme residual interaction. As an example, we consider the spin-dipole (SD) excitations in ^{132}Sn [17]. Results of our calculations for the SD states with different values of the spin and parity for the T_- channel are shown in the Figure. The strength distributions are folded out with a Lorentzian distribution of 1 MeV width. One can conclude that the FRSA can reliably be used for the study of charge-exchange modes.

Let us now discuss the extension of the configuration space to one- and two-phonon terms. As an application of the method, an influence of the 2^+_1-phonon contributing to the coupling terms (4) on the GT strength distribution of ^{78}Ni is evaluated. In particular, we focus on describing the β-decay half-life since this integrated nuclear quantity is sensitive to the inclusion of the $[1^+_i \otimes 2^+_1]$ terms. Its experimentally known value puts an indirect constraint on the calculated GT strength distributions within the Q_β-window. In the allowed GT approximation, the β^--decay rate is expressed by summing the probabilities



Spin-dipole strength distributions in the T_- channel of the ^{132}Sn parent nucleus, calculated by using the SGII force [20]. The results with the FRSA for the p–h interaction (dashed lines), and with the full p–h interaction (solid lines) are shown

of the energetically allowed transitions (in units of $G_A^2/4\pi$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = D^{-1}\left(\frac{G_A}{G_V}\right)^2 \sum_{\nu} f_0(Z, A, -E_\nu) B(GT)_\nu, \tag{9}$$

where $D = 6147$ s and $G_A/G_V = 1.25$ [21]. We employ the Skyrme interaction SLyIII.0.7 [22] which describes isotopic properties of nuclei from the β-stability line to the drip lines. As a result, we find the half-life equal to 370 ms within the RPA. At the same time, taking into account the coupling leads to a decrease of the half-life, $T_{1/2} = 220$ ms. Our results are in reasonable agreement with the experimental value, $(T_{1/2})_{\rm exp} = 110^{+100}_{-60}$ ms [23]. An inclusion of other coupling terms is in progress now.

CONCLUSIONS

A finite rank separable approximation for the charge-exchange QRPA calculations with the Skyrme interactions is extended to take into account the coupling between one- and two-phonon terms in the wave functions of excited states. The suggested approach enables one to perform the calculations in very large configurational spaces. Preliminary results of our studies for the phonon–phonon coupling effect on the β-decay half-life $T_{1/2}$ of ^{78}Ni are reported. There is a clear influence of the 2^+_1 phonon on $T_{1/2}$. A systematical study of the effects of the two-phonon terms on the β^--decay rates is still underway.

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Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/